

## 13.2 and 13.3 (part 1) Overview

- $\mathbf{r}'(t) = \langle x'(t), y'(t), z'(t) \rangle =$  the tangent vector
- $|\mathbf{r}'(t)| = \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2} =$  speed
- $\mathbf{r}''(t) = \langle x''(t), y''(t), z''(t) \rangle =$  acceleration vector
- $\mathbf{T}(t) = \frac{1}{|\mathbf{r}'(t)|} \mathbf{r}'(t) =$  the unit tangent vector
- $\int \mathbf{r}(t) dt = \langle \int x(t) dt, \int y(t) dt, \int z(t) dt \rangle$
- The tangent line to  $\mathbf{r}(t)$  at  $t = t_0$  is given by  
 $x = x_0 + at, y = y_0 + bt, z = z_0 + ct,$   
where  $\langle x_0, y_0, z_0 \rangle = \mathbf{r}(t_0)$ , and  $\langle a, b, c \rangle = \mathbf{r}'(t_0)$ .
- $\int_a^b \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2} dt =$  Arc Length
- $\kappa(t) = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3} =$  curvature.

## Example:

Consider the vector function  $\mathbf{r}(t) = \langle t, \cos(2t), \sin(2t) \rangle$ .

1. Find  $\mathbf{r}'(t)$ ,  $\mathbf{T}(t)$  and  $\mathbf{r}''(t)$ .
2. Find  $\int \mathbf{r}(t) dt$ .
3. Find the equation for the tangent line at  $t = \frac{\pi}{4}$ .
4. Find the arc length from 0 to 3.
5. Reparameterize in terms of arc length.
6. Find the curvature at  $t = 0$ .

### Example Solutions:

1.  $\mathbf{r}'(t) = \langle 1, -2 \sin(2t), 2 \cos(2t) \rangle.$

$$\mathbf{T}(t) = \frac{1}{\sqrt{1+4\sin^2(2t)+4\cos^2(2t)}} \langle 1, -2 \sin(2t), 2 \cos(2t) \rangle = \left\langle \frac{1}{\sqrt{5}}, -\frac{2 \sin(2t)}{\sqrt{5}}, \frac{2 \cos(2t)}{\sqrt{5}} \right\rangle.$$

$$\mathbf{r}''(t) = \langle 0, -4 \cos(2t), -4 \sin(2t) \rangle.$$

2.  $\int \mathbf{r}(t) dt = \left\langle \frac{1}{2}t^2 + C_1, \frac{1}{2} \sin(2t) + C_2, -\frac{1}{2} \cos(2t) + C_3 \right\rangle.$

3.  $\mathbf{r}(\pi/4) = \langle \pi/4, 0, 1 \rangle.$

$$\mathbf{r}'(\pi/4) = \langle 1, -2, 0 \rangle.$$

Thus,  $x = \pi/4 + t$ ,  $y = 0 - 2t$ ,  $z = 1$ .

4.  $\int_0^3 \sqrt{1 + 4 \sin^2(2t) + 4 \cos^2(2t)} dt = \int_0^3 \sqrt{5} dt = 3\sqrt{5}.$

5.  $s(t) = \int_0^t \sqrt{1 + 4 \sin^2(2u) + 4 \cos^2(2u)} du = t\sqrt{5}$ , so  $t = s/\sqrt{5}$ .

Thus,  $\mathbf{r}(s) = \langle s/\sqrt{5}, \cos(2s/\sqrt{5}), \sin(2s/\sqrt{5}) \rangle.$

6.  $\mathbf{r}'(0) = \langle 1, 0, 2 \rangle.$

$$\mathbf{r}''(0) = \langle 0, -4, 0 \rangle.$$

$$|\mathbf{r}'(0)| = \sqrt{1^2 + 0^2 + 2^2} = \sqrt{5}.$$

$$\mathbf{r}'(0) \times \mathbf{r}''(0) = \langle 8, 0, -4 \rangle.$$

$$\text{Thus, } \kappa(0) = \frac{|\mathbf{r}'(0) \times \mathbf{r}''(0)|}{|\mathbf{r}'(0)|^3} = \frac{\sqrt{8^2+0^2+4^2}}{\sqrt{5}^3} = \frac{\sqrt{80}}{\sqrt{5}^3} = \frac{4\sqrt{5}}{\sqrt{5}^3} = \frac{4}{5}.$$